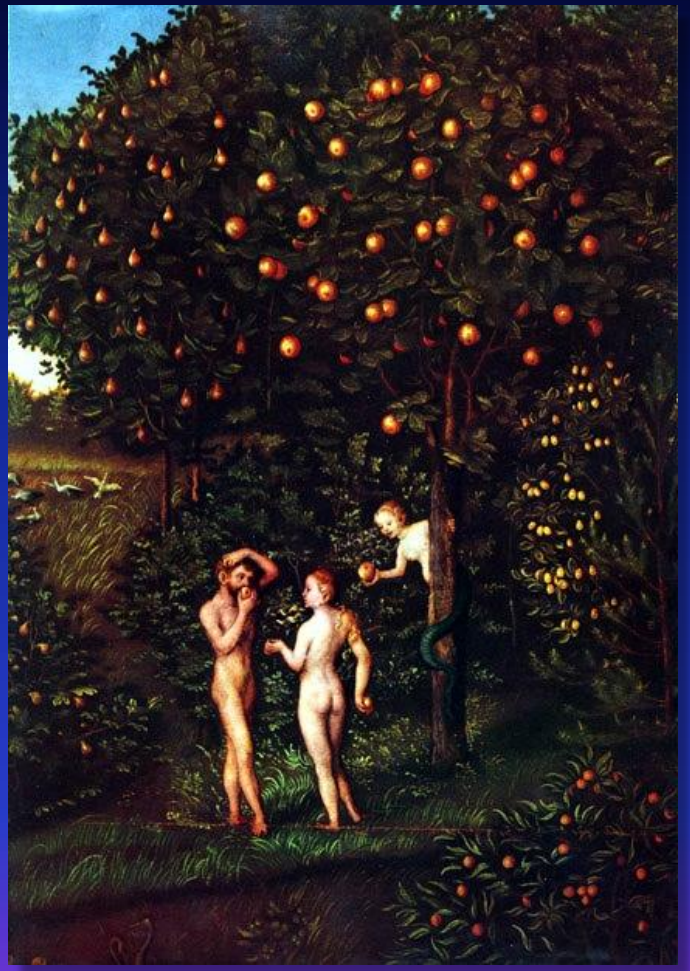
Statistics module 4: Calculating a t score

- ✓ Click “slide show” to start this presentation as a show.
- ✓ Remember: focus & think about each point; do not just passively click.



Cranach, *Tree of Knowledge* [of Good and Evil] (1472)

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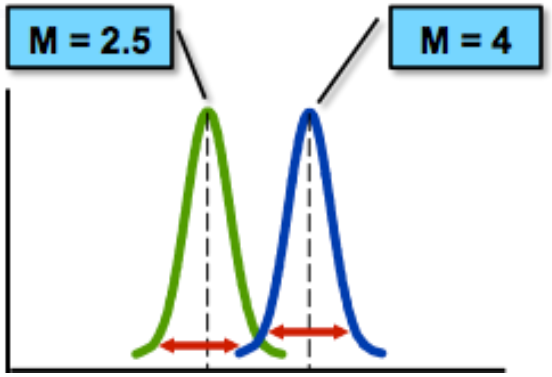

This series has seven modules:

1. Introduction to statistics & number scales
2. The Z score and the normal distribution
3. The logic of research; Plato's Allegory of the Cave
4. Testing hypotheses: The critical ratio
5. Calculating a t score
6. Testing t: The Central Limit Theorem
7. Correlations: Measures of association

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What is our (statistical) hypothesis?

a. **That the mean score (*M*) for the experimental group is greater than** (or less than, depending upon our hypothesis…) **the *M* for the control group…**

b. **…by more than we might expect by chance alone.**

What is the "null" hypothesis?

- ✓ **Any difference between the *M* for the experimental group and the *M* for the control group is by chance alone.**
- ✓ **$M_{experimental} - M_{control} = 0$, except for chance (error variance)**

The research question (in statistical terms):

- ✓ **In our study, is the difference between the group Means** ($M_{exp} - M_{control}$) **greater than** (or less than…) **0 by more than we would expect by chance alone?**

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The concept underlying the t test is the critical ratio:

How strongly did the independent variable affect the outcome?

How much error variance ["uncertainty", "noise"] **is there in the data**

$$\text{Critical ratio} = \frac{\text{Experimental effect}}{\text{Error variance}} = \frac{(M_{exp} - M_{control}) - 0}{\sqrt{\frac{\text{Variance}_{exp}}{n_{exp}} + \frac{\text{Variance}_{control}}{n_{control}}}} = t$$

For a t-test:

The experimental effect is the difference between the *M*s of the experimental & control groups

The error variance is the square root of the summed variances of the groups, similar to a two-group standard deviation.

Critical ratio = Experimental effect / Error variance

$$t = \frac{\text{Difference between groups}}{\text{standard error of M}} = \frac{(M_{group1} - M_{group2}) - 0}{\sqrt{\frac{Variance_{grp1}}{n_{grp1}} + \frac{Variance_{grp2}}{n_{grp2}}}}$$

$$t = \frac{(M_{group1} - M_{group2}) - 0}{\sqrt{\frac{SS_{grp1}/df_{grp1}}{n_{grp1}} + \frac{SS_{grp2}/df_{grp2}}{n_{grp2}}}}$$

- How strong is the experimental effect?
- How much error variance is there

$$t = \frac{(M_{group1} - M_{group2}) - 0}{\sqrt{\frac{\sum(X - M)^2_{grp1}/n - 1_{grp1}}{n_{grp1}} + \frac{\sum(X - M)^2_{grp2}/n - 1_{grp2}}{n_{grp2}}}}$$

Critical ratio = Experimental effect / Error variance

$$t = \frac{\text{Difference between groups}}{\text{standard error of M}} = \frac{(M_{group1} - M_{group2}) - 0}{\sqrt{\frac{Variance_{grp1}}{n_{grp1}} + \frac{Variance_{grp2}}{n_{grp2}}}}$$

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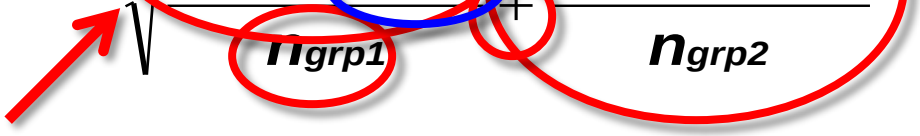

Standard error:

- Calculate the variance for for group 1
  - Sum of squares
  - Divided by degrees of freedom (n-1)
- Divide by n for group 1
- Repeat for group 2
- Add them together
- Take the square root

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Critical ratio = Experimental effect / Error variance

$$t = \frac{\text{Difference between groups}}{\text{standard error of M}} = \frac{(M_{group1} - M_{group2}) - 0}{\sqrt{\frac{Variance_{grp1}}{n_{grp1}} + \frac{Variance_{grp2}}{n_{grp2}}}}$$

$$t = \frac{(M_{group1} - M_{group2}) - 0}{\sqrt{\frac{SS_{grp1}/df_{grp1}}{n_{grp1}} + \frac{SS_{grp2}/df_{grp2}}{n_{grp2}}}}$$

The expanded version…

$$t = \frac{(M_{group1} - M_{group2}) - 0}{\sqrt{\frac{\sum(X - M)^2_{grp1}/n - 1_{grp1}}{n_{grp1}} + \frac{\sum(X - M)^2_{grp2}/n - 1_{grp2}}{n_{grp2}}}}$$

Compute a t score

Critical ratio = Experimental effect / Error variance

$$t = \frac{(M_{group1} - M_{group2}) - 0}{\sqrt{\frac{\sum(X - M)^2_{grp1} / n - 1_{grp1}}{n_{grp1}} + \frac{\sum(X - M)^2_{grp2} / n - 1_{grp2}}{n_{grp2}}}}$$

Compute the Experimental Effect:

- ✓ Calculate the Mean for each group, subtract $Mgroup_2$ from $Mgroup_1$.

Compute the Standard Error

- ✓ Calculate the variance for each group

Calculate the Variance using the box method:

X	M	X - M	$(X - M)^2$
7	4	3	9
6	4	2	4
2	4	-2	4
1	4	-3	9
4	4	0	0
1	4	-3	9
7	4	3	9
4	4	0	0
2	4	-2	4
6	4	2	4
n = 10		Σ = 0	Σ = 52
Σ= 40			
M = 40/10 = 4			

1. Enter the **Scores**.

2. Calculate the **Mean.**

3. Calculate **Deviation scores:**
   → Simple deviations: Σ (X – M) = 0
   → **Square the deviations** to create + values:
   Σ Squares = $\Sigma(X - M)^2 = 52$

4. Degrees of freedom:
   df = [n – 1] = [10 – 1] = 9

5. Apply the **Variance** formula:

$$s^2 = \frac{\text{å } (X-M)^2}{df} = \frac{52}{9} = 5.8$$

Compute a t score

Critical ratio = Experimental effect / Error variance

$$t = \frac{(M_{group1} - M_{group2}) - 0}{\sqrt{\frac{\sum(X - M)^2_{grp1}}{n - 1_{grp1}} + \frac{\sum(X - M)^2_{grp2}}{n - 1_{grp2}}}}$$

n_{grp1} n_{grp2}

→ effect

→ error

Compute the Experimental Effect:

- ✓ Calculate the Mean for each group, subtract group$_2$ M from group$_1$ M.

Compute the Standard Error

- ✓ Calculate the variance for each group
- ✓ Divide each variance by n for the group
- ✓ Add those computations
- ✓ Take the square root of that total

Compute t

- ✓ Divide the Experimental Effect by the Standard Error

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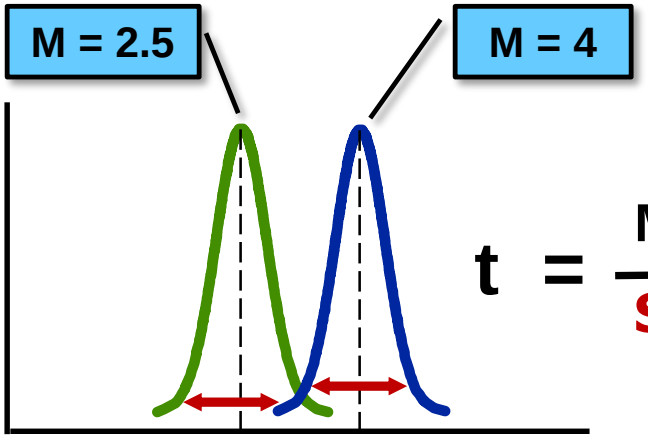
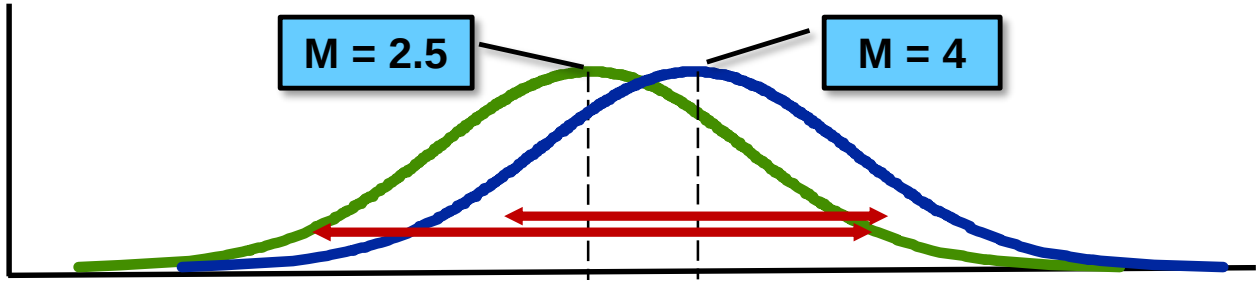
Examples of deriving t values



$$t = \frac{M_1 - M_2 = 4 - 2.5 = 1.5}{\text{Standard error} = .75} = \frac{1.5}{.75} = 2$$

$$t = \frac{M_1 - M_2 = 4 - 2.5 = 1.5}{\text{Standard error} = 1.75} = \frac{1.5}{1.75} = .86$$



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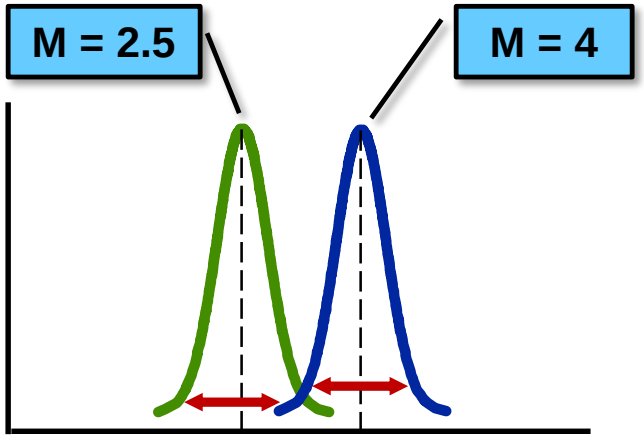

Why does this have a t value = 2?

a. **The variance within each group is large relative to the difference between the group means.**

b. **The M of the larger group = 4 and there are 2 groups**

c. **The difference between the group means is large relative to the variance within each group**

d. **t is a random number**

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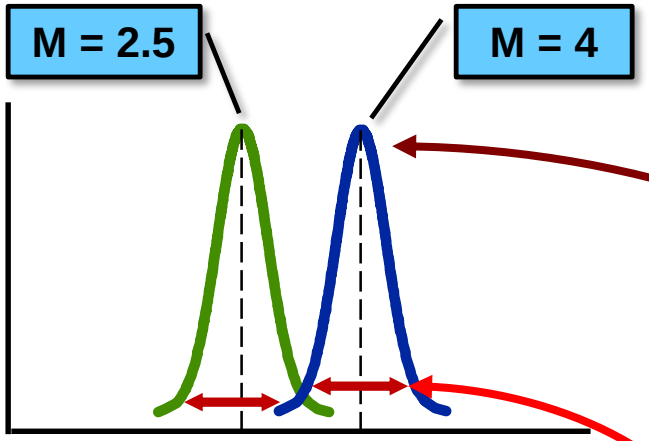

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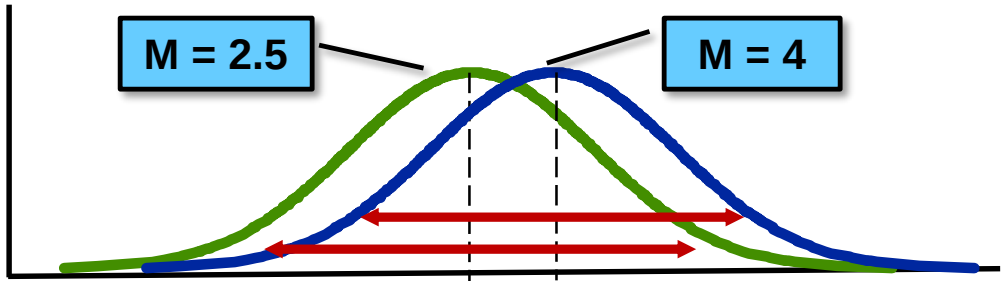
d. **t is a random number**

Clicker, 2



Why does this have a t value = .86?

a. **The variance within each group is large relative to the difference between the group means.**

b. **The M of the larger group = 4 and there are 2 groups**

c. **The difference between the group means is large relative to the variance within each group**

d. **t is a random number**

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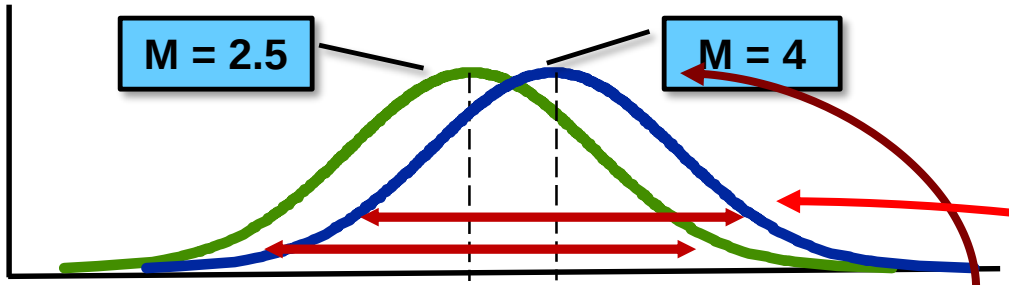
Clicker, 2



Why does this have a t value = .86?

a. The variance within each group is large relative to the difference between the group means.

b. The M of the larger group = 4 and there are 2 groups

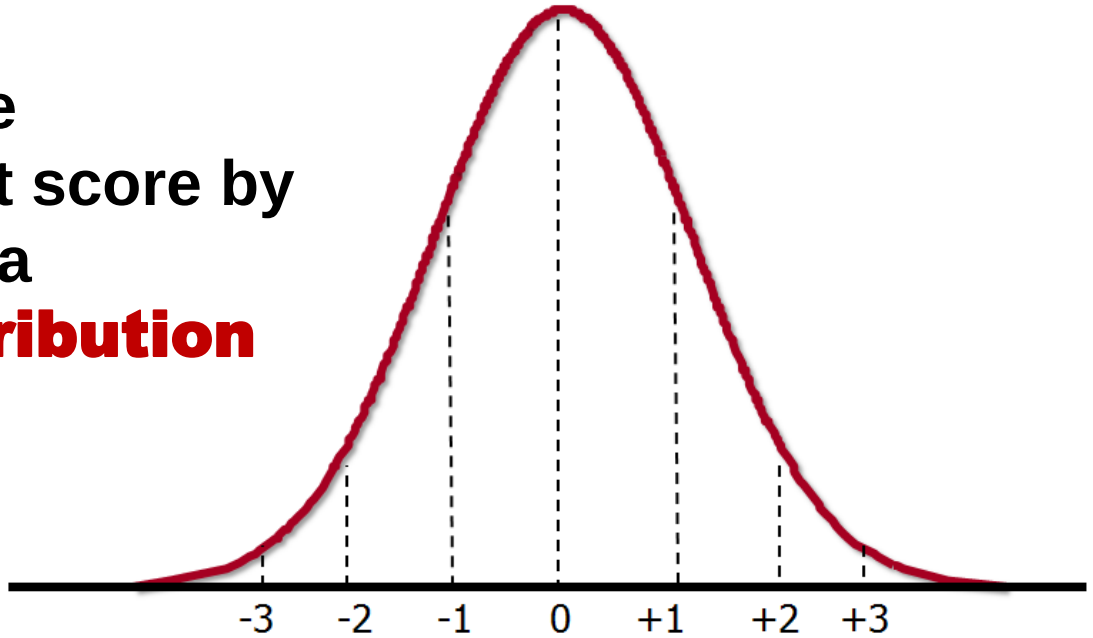
c. The difference between the group means is large relative to the variance within each group

d. t is a random number

Sampling distribution & statistical significance

- Any 2 group *M*s differ at least slightly by chance.
- Any t score is therefore > 0 or < 0 by chance alone.
- We assume that a *t* score with less than 5% probability of occurring [$p < .05$] is ***not*** by chance alone

- We calculate the probability of a t score by comparing it to a **sampling distribution**



Sampling distribution of t scores

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The Sampling Distribution

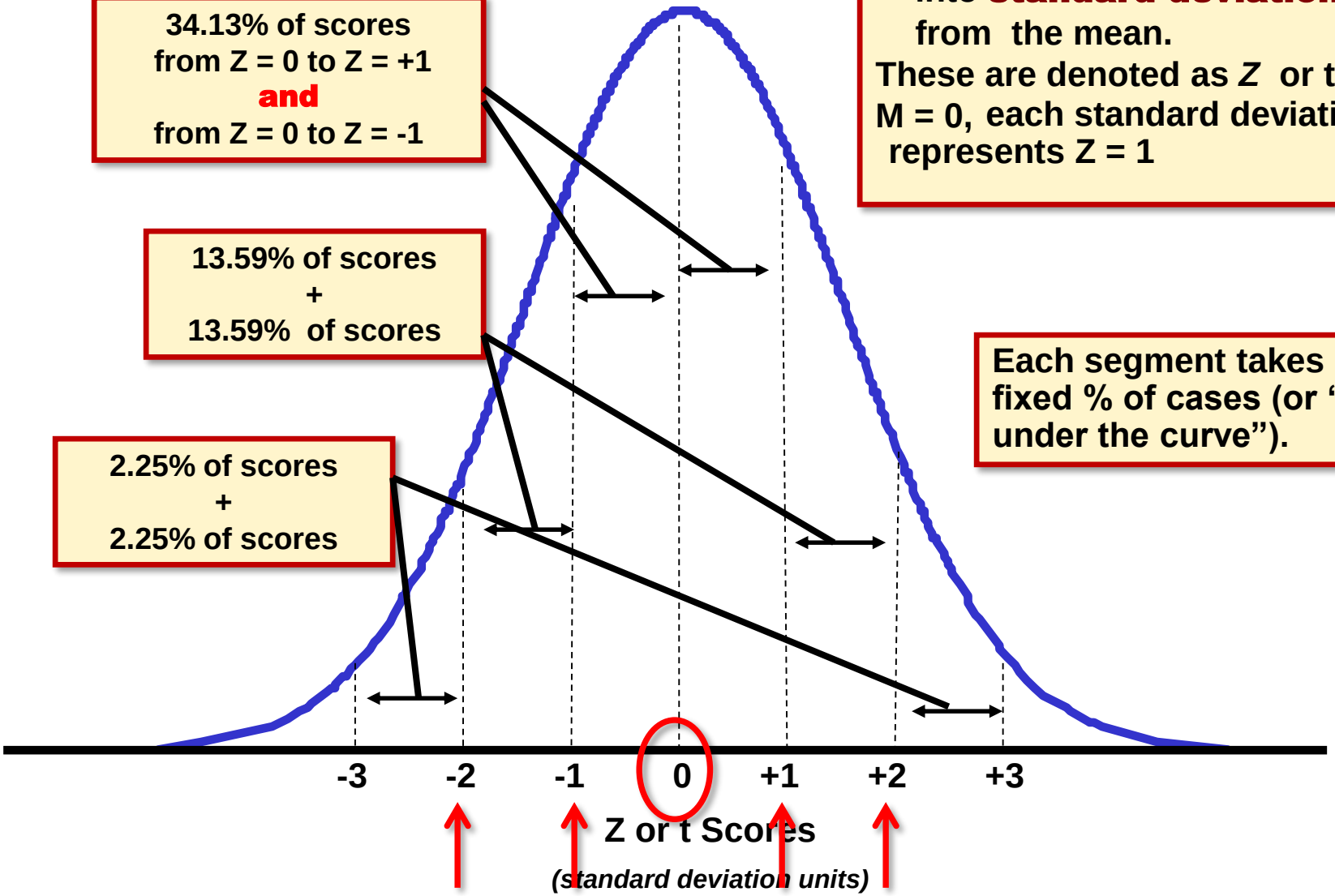
We can segment the population into **standard deviation units** from the mean.
These are denoted as *Z* or t
M = 0, each standard deviation represents Z = 1

34.13% of scores
from Z = 0 to Z = +1
and
from Z = 0 to Z = -1

13.59% of scores
+
13.59% of scores

2.25% of scores
+
2.25% of scores

Each segment takes up a fixed % of cases (or "area under the curve").



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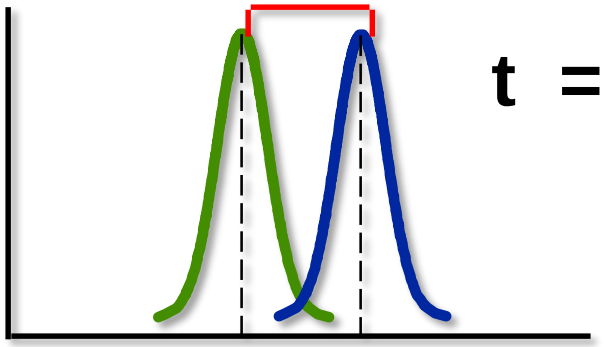
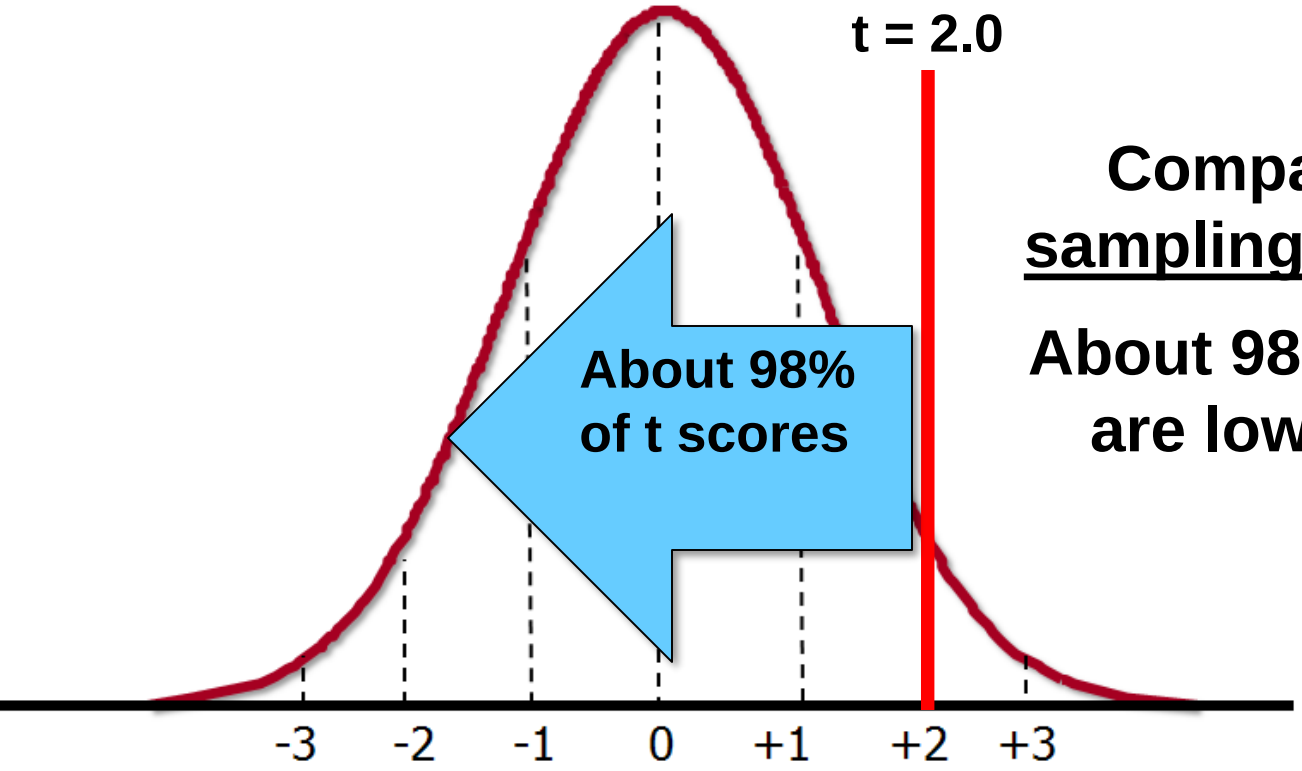
t scores and statistical significance, 1



$$t = \frac{M_1 - M_2 = 4 - 2.5}{\text{Standard error}} = \frac{1.5}{.75} = 2$$



Sampling distribution of t scores

Comparing t to a sampling distribution:

About 98% of t values are lower than 2.0

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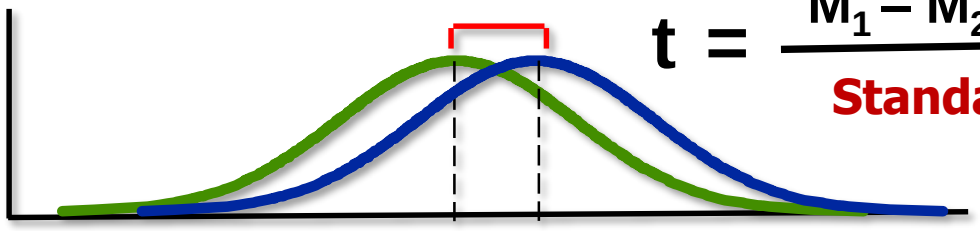
t scores and statistical significance, 1



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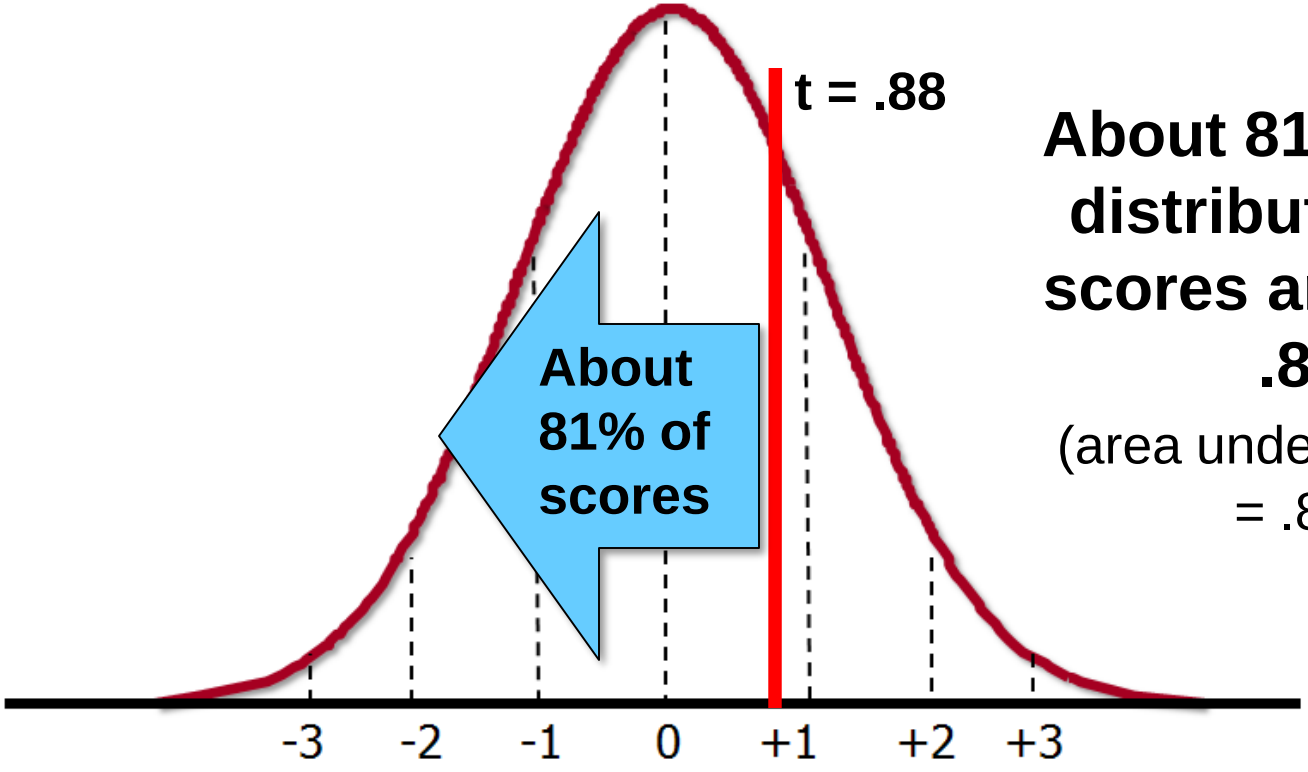

Sampling distribution of t scores

About 81% of the distribution of t scores are below .88

(area under the curve = .81)

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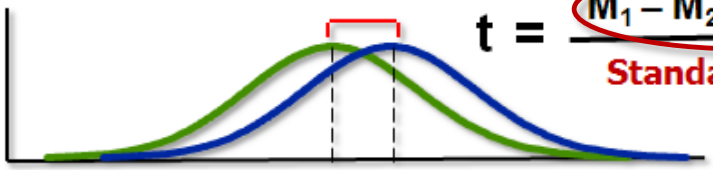
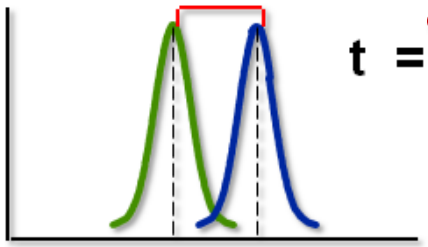
Between v. within group variance: t-test logic



$$t = \frac{M_1 - M_2 = 4 - 2.5}{\text{Standard error}} = \frac{1.5}{1.75} = .86$$

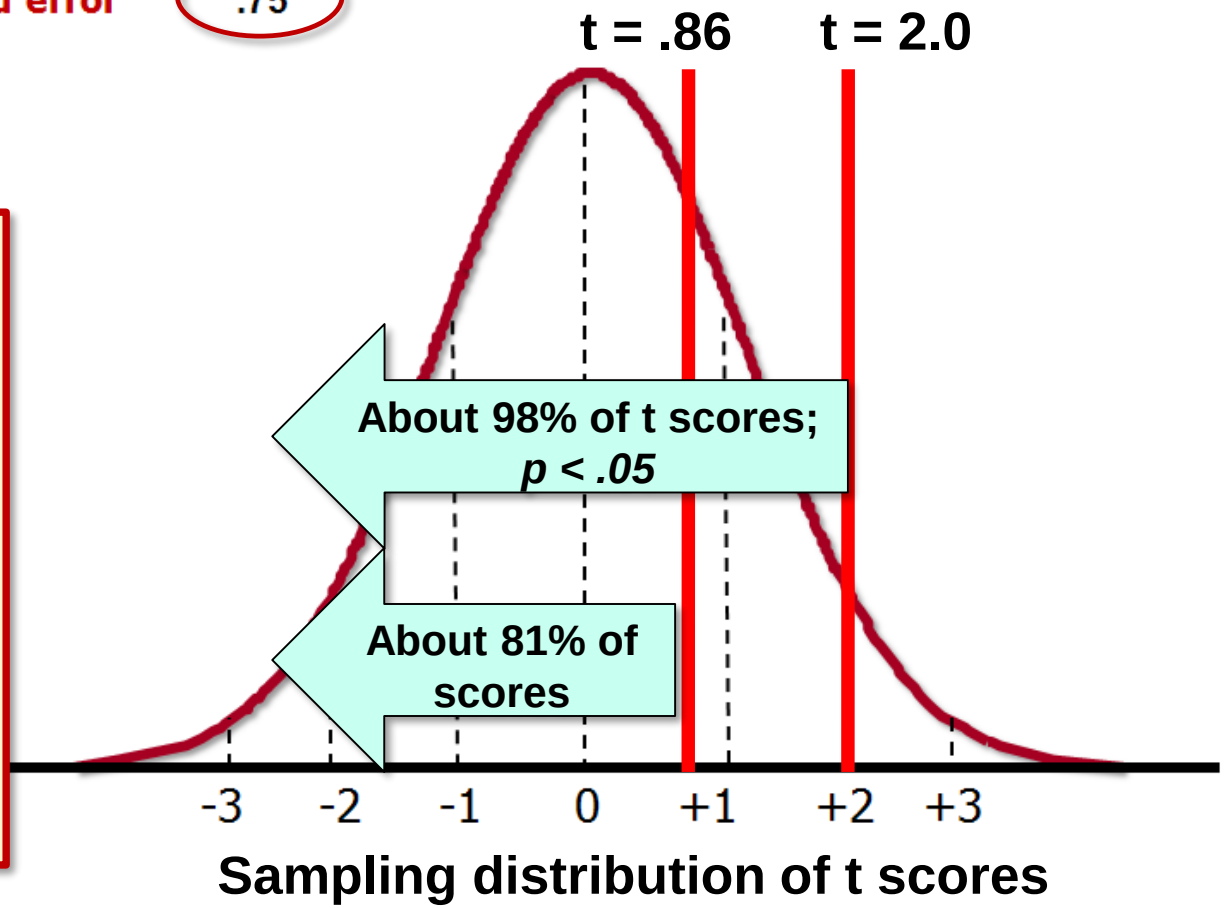
The difference between *M*s is the same in the two data sets.



$$t = \frac{M_1 - M_2 = 4 - 2.5}{\text{Standard error}} = \frac{1.5}{.75} = 2$$

Since the variances differ…

- ✓ **We get different t values**
- ✓ **We make differ judgments about whether these t scores occurred by chance.**



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Continue…

Continue this series by clicking on the module for The Central Limit Theorem.